

CANDIDATE
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FURTHER MATHEMATICS

9231/23

Paper 2

October/November 2018

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value is necessary, take the acceleration due to gravity to be 10 m s^{-2} .

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **22** printed pages and **2** blank pages.



In the collision, the speed of A is halved and its direction of motion is reversed.

(ii) Find the value of e . [2]

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(iii) For this collision, find the ratio of the loss of kinetic energy of A to the loss of kinetic energy of B . [3]

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5 The fixed points A and B are on a smooth horizontal surface with $AB = 2.6$ m. One end of a light elastic spring, of natural length 1.25 m and modulus of elasticity λ N, is attached to A . The other end is attached to a particle P of mass 0.4 kg. One end of a second light elastic spring, of natural length 1.0 m and modulus of elasticity 0.6λ N, is attached to B ; its other end is attached to P . The system is in equilibrium with P on the surface at the point E .

(i) Show that $AE = 1.4$ m. [4]

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The particle P is now displaced slightly from E , along the line AB .

(ii) Show that, in the subsequent motion, P performs simple harmonic motion. [5]

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(iii) Given that the period of the motion is $\frac{1}{7}\pi$ s, find the value of λ . [3]

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6 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{80} \left(3\sqrt{x} - \frac{8}{\sqrt{x}} \right) & 4 \leq x \leq 16, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X . [3]

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The random variable Y is defined by $Y = \sqrt{X}$.

(ii) Find the probability density function of Y . [3]

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7 The random variable T is the lifetime, in hours, of a particular type of battery. It is given that T has a negative exponential distribution with mean 500 hours.

(i) Write down the probability density function of T . [1]

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(ii) Find the probability that a randomly chosen battery of this type has a lifetime of more than 750 hours. [3]

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(iii) Find the median value of T . [3]

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8 The weekly salaries of employees at two large electronics companies, *A* and *B*, are being compared. The weekly salaries of an employee from company *A* and an employee from company *B* are denoted by $\$x$ and $\$y$ respectively. A random sample of 50 employees from company *A* and a random sample of 40 employees from company *B* give the following summarised data.

$$\Sigma x = 5120 \quad \Sigma x^2 = 531\,000 \quad \Sigma y = 3760 \quad \Sigma y^2 = 375\,135$$

(i) The population mean salaries of employees from companies *A* and *B* are denoted by $\$\mu_A$ and $\$\mu_B$ respectively. Using a 5% significance level, test the null hypothesis $\mu_A = \mu_B$ against the alternative hypothesis $\mu_A \neq \mu_B$. [8]

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(ii) State, with a reason, whether any assumptions about the distributions of employees' salaries are needed for the test in part **(i)**. [1]

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- 9 There are a large number of students at a particular college. The heights, in metres, of a random sample of 8 students are as follows.

1.75 1.72 1.62 1.70 1.82 1.75 1.68 1.84

You may assume that heights of students are normally distributed.

- (i) Test, at the 5% significance level, whether the population mean height of students at this college is greater than 1.70 metres. [7]

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(ii) Find a 95% confidence interval for the population mean height of students at this college. [3]

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10 For a random sample of 10 observations of pairs of values (x, y) , the equation of the regression line of y on x is $y = 1.1664 + 0.4604x$. It is given that

$$\Sigma x^2 = 1419.98 \quad \text{and} \quad \Sigma y^2 = 439.68.$$

The mean value of y is 6.24.

(i) Find the equation of the regression line of x on y .

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(ii) Find the product moment correlation coefficient.

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(iii) Test at the 5% significance level whether there is evidence of positive correlation between the two variables. [4]

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OR

A machine is used to produce metal rods. When the machine is working efficiently, the lengths, x cm, of the rods have a normal distribution with mean 150 cm and standard deviation 1.2 cm. The machine is checked regularly by taking random samples of 200 rods. The latest results are shown in the following table.

Interval	$146 \leq x < 147$	$147 \leq x < 148$	$148 \leq x < 149$	$149 \leq x < 150$
Observed frequency	1	2	23	52
	$150 \leq x < 151$	$151 \leq x < 152$	$152 \leq x < 153$	$153 \leq x < 154$
	69	36	15	2

As a first check, the sample is used to calculate an estimate for the mean.

- (i) Show that an estimate for the mean from this sample is close to 150 cm. [2]

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As a second check, the results are tested for goodness of fit of the normal distribution with mean 150 cm and standard deviation 1.2 cm. The relevant expected frequencies, found using the normal distribution function given in the List of Formulae (MF10), are shown in the following table.

Interval	$x < 147$	$147 \leq x < 148$	$148 \leq x < 149$	$149 \leq x < 150$
Observed frequency	1	2	23	52
Expected frequency	1.24	8.32	30.94	59.50
	$150 \leq x < 151$	$151 \leq x < 152$	$152 \leq x < 153$	$153 \leq x$
	69	36	15	2
	59.50	30.94	8.32	1.24

- (ii) Show how the expected frequency for $151 \leq x < 152$ is obtained. [3]

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(iii) Test, at the 5% significance level, the goodness of fit of the normal distribution to the results.

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